(NASA-CR-155148) FEASIBILITY ANALYSIS OF GRAVITATIONAL EXPERIMENTS IN SPACE Final Report (Stanford Univ.) 35 p HC A03/MF A01 CSCL 22A

N77-33223

Unclas
G3/12 49470

High-Energy Physics Laboratory

W. W. Hansen Laboratories of Physics

STANFORD UNIVERSITY

STANFORD, CALIFORNIA

Final Report on Contract No. 954524

FEASIBILITY ANALYSIS OF GRAVITATIONAL EXPERIMENTS IN SPACE

Principal Investigator:
C. W. F. Everitt

W. W. Hansen Laboratories of Physics
Stanford University
Stanford, California 94305

September 1977

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

A. INTRODUCTION

This report deals with several experiments on gravitation and General Relativity suggested by different workers in the past ten or more years, examines their feasibility, and reviews the advantages, if any, of performing them in space. The experiments include (1) the Gyro Relativity experiment, (2) experiments to test the equivalence of gravitational and inertial mass, (3) an experiment suggested by P. K. Chapman to look for non-geodesic motion of spinning bodies in orbit around the Earth, (4) experiments to look for changes of the gravitational constant G with time, (5) a variety of suggestions due to Braginsky, Caves and Thorne for "laboratory" tests of experimental gravity, (6) gravitational wave experiments.

Tests of General Relativity may be divided into two classes: those involving motions of massive bodies and those involving effects on electromagnetic radiation. Of the three "classical tests" suggested by Einstein in 1915: (1) the relativistic correction to the precession of the perihelion of the planet Mercury, (2) the gravitational redshift, (3) the deflection of starlight by the Sun, only the precession of the perihelion of Mercury checks Einstein's equations of motion through measuring an effect on a massive body. A fourth consequence of the theory which attracted much attention during the 1920's through the observations of Hubble was the concept of an expanding Universe; but this result, exciting as it was for cosmology, cannot be taken as evidence for General Relativity, since it is, as E. A. Milne pointed out in a noteworthy though little known paper of the 1930's, also to be expected on a Newtonian interpretation of the Universe. Picture an exploding body sending out pieces of material at different velocities in all directions. After time t the particles moving with velocity v will have travelled a distance $\ell = vt$; the velocity $v(\ell)$ at a distance ℓ from the origin will be proportional to ℓ in exact account with Hubble's discovery.

The starlight deflection amounts to 1.7 arc-sec at the rim of the Sun; the perihelion precession is a residuum of 0.43 arc-sec/year after much larger Newtonian effects have been calculated out. The extreme difficulty in testing General Relativity arises because the deviations from Newtonian gravitation near a body of mass M and radius R are characterized by the parameter GM/c^2R, where G is the gravitational constant and c the velocity of light, in contrast to special relativistic effects, which are characterized by the parameter v/c, where v is the relative velocity of two bodies. For small particles in the laboratory v/c may easily approach unity, but GM/c^2R at the surface of a body of density ρ and radius r_o amounts to $2.8 \times 10^{-28} \rho r_o^2$. For a laboratory object of diameter 1 m and density 10 this quantity is 7×10^{-24} and it remains small even on the scale of the solar system, being about 10^{-9} for the Earth and 10^{-6} for the Sun. The Sun, relativistically speaking, is a small body.

The status of the classical tests of relativity has been extensively discussed. The best measure of starlight deflection is from observations on the radio source 3C279 which lies in the ecliptic and is occulted by the Sun each year in October. Recent data show agreement with General Relativity to within the experimental uncertainty of 1%. A related effect pointed out by I. Shapiro in 1963 is the time delay in radar ranging measurements to planets or spacecraft passing behind the Sun. This delay has been observed many times; the best results after correcting for many subsidiary effects agree with Einstein's theory to about 2%. The effect on the orbit of Mercury remains in many ways the most impressive test of General Relativity. Its first impact was dramatic because the predicted effect exactly matched the mysterious anomaly in the perihelion shift discovered in the 1850's by Leverrier.* A weak point concerns the effects of the Sun's quadrupole mass-moment. In 1964 R. H. Dicke, following his work with C. Brans on the Jordan scalar-tensor theory of gravitation, suggested that the interior of

*Leverrier's original estimate of the anomaly was 38 arc-sec/century, but this had been corrected to 42 arc-sec/century by 1915.

the Sun might be rotating more rapidly than its surface and that as a result the Sun might have a quadrupole moment large enough to account for 10% of the Leverrier anomaly, leaving a discrepancy between the observed effect and Einstein's theory. The observations of H. M. Goldenberg and R. H. Dicke in 1968 on the optical shape of the Sun seemed to confirm this. Recent measurements by H. E. Hill and his collaborators disagree with the Goldenberg-Dicke result. However the Sun's quadrupole mass moment has never been measured and the hypothesis that the Sun has a rapidly rotating inner core is not implausible.

The gravitational redshift is not a test of General Relativity per se, but of one of the most important hypotheses underlying the theory: the so-called "strong" equivalence principle. First some comment on the equivalence principle itself. In gravitation, it has two aspects commonly, but rather misleadingly, called "strong" and "weak" equivalence. "Weak" equivalence means that the gravitational and inertial masses of bodies of different composition are identical. The famous experiment, popularly attributed to Galileo, of dropping balls of lead and wood from the Leaning Tower of Pisa and seeing whether they fell with equal times, was a test of "weak" equivalence. Newton made the first good experiment by timing the periods of pendulums with balls of different materials and verified the equivalence of gravitational and inertial mass to 1 part in 10^3. A great step forward was made towards the end of the 19th century by R. Eötvös using a torsion balance from which were suspended two masses of different materials. When sitting in the laboratory the two masses are subject to the gravitational attraction of the Earth and also to a centrifugal acceleration of about 1.4 cm/sec^2 due to the Earth's rotation. When the torsion head is turned through 180° the two masses change place; the directions relative to the torsion arm of the centrifugal acceleration and the component of the Earth's gravitation balancing it reverse; hence if there is any difference between the ratios of gravitational and inertial mass for the two bodies, the torsion arm

will turn through an angle slightly different from the 180° rotation of the torsion head, and the difference in angle is a measure of the quantity now usually called the Eötvös ratio

$$\eta = \frac{\left(\frac{m}{M}\right)_A - \left(\frac{m}{M}\right)_B}{\frac{1}{2}\left[\left(\frac{m}{M}\right)_A + \left(\frac{m}{M}\right)_B\right]} \tag{1}$$

where m and M are the inertial and gravitational masses of the two bodies A and B. Eötvös, Pekar and Fekete established the equivalence of gravitational and inertial mass for a wide variety of materials to a few parts in 10^8. A variant experiment suggested by Eötvös was to look at a daily oscillation of the torsion balance turning with the Earth due to its change in orientation with respect to the gravitational field of the Sun. The effect is smaller than that from the Earth's centrifugal acceleration since the Sun's acceleration at the Earth's orbit is 0.6 cm/sec^2 rather than 1.4 cm/sec^2, but the method allows one to avoid turning the torsion head and hence avoid errors due to elastic hysteresis in the torsion balance. An experiment of this kind was done by Roll, Krotkov and Dicke in 1960. They demonstrated the equivalence of gravitational and inertial mass for gold and aluminum test bodies to 3 parts in 10^{11}. A similar experiment with platinum and aluminum was performed by Braginsky and Panov in 1975. They claimed an accuracy of 1 part in 10^{12}, but for reasons discussed elsewhere I am persuaded that the Roll-Krotkov-Dicke experiment remains the most accurate test of the weak equivalence principle performed so far.

The "strong" equivalence principle is the further hypothesis introduced by Einstein in 1911 that a gravitational acceleration is equivalent to an inertial acceleration in all respects, and therefore that light moving in a gravitational field will experience a change in frequency identical to the change in frequency calculated from special relativity for light in an accelerating inertial frame. The result is a blue or red shift

depending on whether the energy of the photons is increased or decreased by the change in gravitational potential. During the 1930's gravitational redshifts in stars were studied. They turned out to be mixed up with other effects which limited the accuracy of the gravitational test to about 20%. A good check only became possible in 1960 when Pound and Rebka developed laboratory techniques to measure the redshift by means of the Mössbauer effect. In 1965 Pound and Snyder performed a measurement which confirmed the strong equivalence principle to 1%. A further advance, and one of the most beautiful gravitational experiments done to date, has been the sub-orbital Scout launch in June 1976 of a hydrogen maser clock by R. C. Vessot and his colleagues of The Smithsonian Astronomical Observatory in cooperation with NASA Marshall Center. Data reduction is still in progress. So far the results confirm the strong equivalence principle to 1 part in 10^4. More analysis should reduce the experimental uncertainties to 20 parts per million.

The "weak" and "strong" equivalence principles exemplify the distinction made above between gravitational experiments based on studying motions of massive bodies and those based on the effects of gravitation on electromagnetic radiation. Recognizing this and detaching our minds from both the sequences of history and the charm of Einstein's imagination, we may properly ask whether the loaded adjectives "weak" and "strong" accurately convey the relative importance of the two kinds of tests. In reality the apparent identity between gravitational and inertial mass remains one of the deepest mysteries of physics, and is, if anything, even more astonishing than the frequency shift predicted by Einstein.

Most of the experiments to be reviewed in this report comprise searches for new gravitational effects on massive bodies. The Gyro Relativity experiment measures the gravitational spin-orbit and spin-spin interactions between the Earth and a gyroscope

in orbit around it. The orbiting equivalence principle experiments are tests of the "weak" equivalence principle. The experiments to measure the change of gravitational constant with time depend on masses suspended from a torsion balance or on timing of the period of two masses in orbit around one another. The experiment to search for "non-geodesic" motion depends on attempting to measure the differential acceleration on two counter-rotating hoops in orbit round the Earth. The Braginsky-Caves-Thorne experiment to look for gravitational spin-spin coupling seeks to measure the varying gravitational attraction between a spinning body suspended from a torsion balance of several hours period and a large rotating flywheel whose speed is modulated with a period resonant with the torsion balance. Gravitational wave experiments use a large aluminum or sapphire bar as the gravitational counterpart to the Hertzian dipole antenna, or alternatively make laser interferometer measurements on extended structures.

The experiments listed above are in very different stages of development. Much the most advanced is the Gyro Relativity experiment, which is now ready for Phase B study. Others are being actively worked on in the laboratory, while others are little more than fledgling ideas. In this study, instead of a step by step description and theoretical analysis of each, I propose to select some crucial issues they have in common and illustrate them by reference to particular experiments as I go along. Although this will break up the description of the experiments themselves, it will have the advantage of giving the reader unfamiliar with gravitational experiments some idea of what to look out for when new proposals come his way.

Section B discusses the potential advantages of space operation for gravitational experiments. Many of the experiments utilize cryogenic techniques. The reasons, good and bad, that have been suggested for applying cryogenic techniques are discussed in Section C. In space, the effects of gravity gradients

on the test bodies are often of critical importance. These are discussed in Section D with special reference to the Gyro Relativity experiment, equivalence principle experiments and the experiment on non-geodesic motion. In some experiments the disturbances from gravity gradients can be avoided by using an Earth-oriented orbiting laboratory. These are discussed in Section D.

B. POTENTIAL ADVANTAGES OF SPACE AS AN ENVIRONMENT FOR EXPERIMENTS ON GRAVITATION

Space offers three potential advantages over an Earth-based laboratory as an environment for new experiments on gravitation.

(1) Freedom from seismic noise

(2) An environment where test masses are nearly in free fall rather than requiring support against the 1-g acceleration of Earth

(3) The possibility in particular experiments of using the Earth, the Sun or Jupiter as the source of the gravitational field generating some effect.

Take as an example the Gyro Relativity experiment. According to the calculations of Schiff an ideal torque free gyroscope in free fall about a rotating massive sphere such as the Earth undergoes a relativistic precession with respect to the framework of the fixed stars given by

$$\Omega = \frac{3GM}{2c^2R^3}\ (\underline{R} \wedge \underline{v}) + \frac{GI}{c^2R^3}\left[\frac{3\underline{R}}{R^2}\ (\underline{\omega}_e \cdot \underline{R}) - \underline{\omega}_e\right] \qquad (2)$$

where $\underline{R}$ and $\underline{v}$ are the coordinate and velocity of the gyroscope, and M, I and $\underline{\omega}_e$ are the mass, moment of inertia and angular velocity of the central body. The first term gives the spin-orbit, or geodetic, precession Ω^G due to motion of the gyroscope through the gravitational field; the second gives the spin-spin, or motional, precession Ω^M due to rotation of the central body. For gyroscopes in polar orbit the two effects are at right angles as in Figure 1 and have (in General Relativity) integrated values of 6.9 arc-sec/year for Ω^G and 0.050 arc-sec/year for Ω^M at an

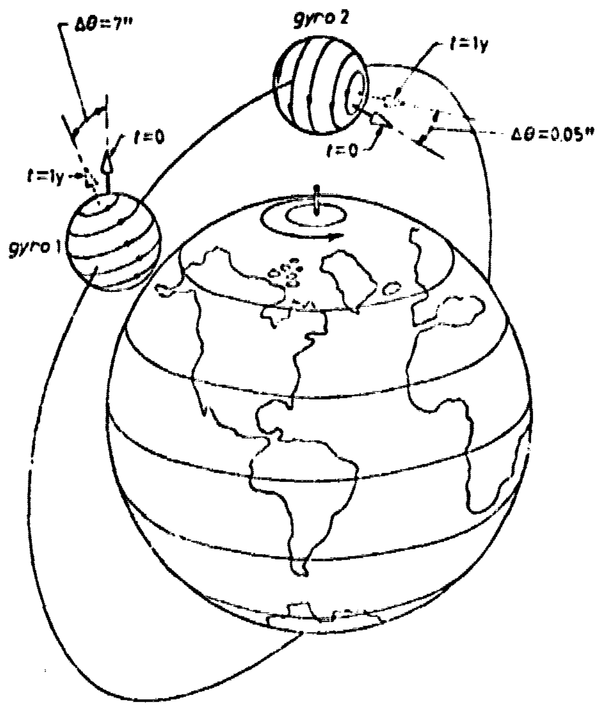

Figure 1: Relativistic motions of gyroscopes

orbit altitude of 400 nautical miles. The gyroscope for the experiment is illustrated in Figure 2. It consists of a ball 4 cm in diameter made from optically selected fused quartz coated with a thin film of superconductor. The ball is

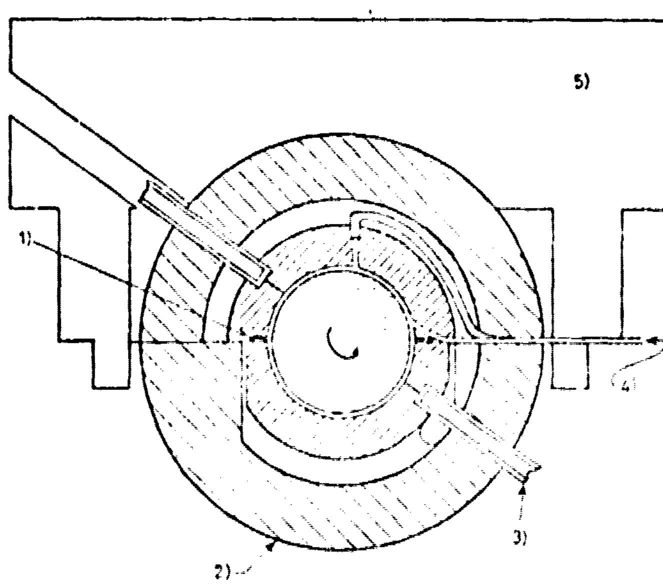

Figure 2: Gyroscope for Schiff Gyro Relativity experiment:
1) readout ring, 2) superconducting magnetic shield, 3) leads to support electrodes, 4) inlet and exhaust for gas spin up, 5) mounting ring

electrically suspended within a spherical quartz housing by voltages applied to three mutually perpendicular sets of condenser plates. It is spun up initially to a speed of about 200 Hz by gas jets, after which the gas is pumped out and the ball is allowed to run freely in a vacuum of about 10^{-9} torr. The entire device is surrounded by a spherical superconducting magnetic shield. The general principles of the gyroscope are based on those of the electrically suspended gyroscopes developed by Honeywell Incorporated, following the work of the late A. Nordsieck and his collaborators at the University of Illinois. The support electrodes are circular pads 2 cm in diameter and 4×10^{-3} cm from the rotor. The suspension system applies 20 kHz alternating voltages to each electrode; when the gyroscope experiences an acceleration parallel to an electrode axis, the voltage is raised on one plate and lowered on the other to keep the ball centered. Since the ball is not perfectly round the suspension voltages exert torques on it. A 1 MHz sensing voltage measures the ball position; this too exerts forces and therefore torques on the rotor.

The enormous advantage of space as an environment for the Gyro Relativity experiment becomes clear upon examining the suspension and mass unbalance torques on the gyro rotor. The suspension torques due to out of roundness of the ball scale as the square of the support voltage. On Earth about 2 kV has to be applied across the 4×10^{-3} cm gap to suspend the ball. In space the gyro is in nearly free fall and the support potential can be reduced to about 0.5 V: a reduction of seven orders of magnitude in the torque, or actually more than seven orders since the averaging of the torques is also improved. Now consider mass unbalance. The drift rate of a gyroscope from some extraneous nonrelativistic torque Γ applied at right angles to its spin axis is given by $\Omega = \Gamma/I\omega_s$. In the presence of a transverse acceleration f a rotor that was perfectly spherical and supported about its center of geometry but made of a material having density variations of magnitude $\Delta\rho$ would experience a mass-unbalance drift

Ω^u given by

$$\Omega^u = \frac{C_u}{2} \frac{\Delta\rho}{\rho} \frac{f}{v_s} \tag{3}$$

where v_s is the peripheral velocity of the ball and C_u is a constant lying between 0 and 1 specifying the symmetry of the density variations. To make a gyro capable of attaining a residual drift rate below some design goal Ω_0 (10^{-16} rad/sec for the Gyro Relativity experiment) the product of average residual acceleration and density variations must satisfy the inequality $f\Delta\rho < 2\rho v_s \Omega_0 / C_u$, which means for the actual gyroscope $f\Delta\rho < 2.5 \times 10^{-12}$ cm/sec^2. On Earth the density variations in the material could not be allowed to exceed 1 part in 10^{15}, which is absurd. In space they can be 1 part in 10^6.

Another advantage of the absence of gravity is in the stability of the reference telescope. On Earth the telescope cantilevered from a mounting ring sags under its own weight through 0.13 arc-sec, and is also subject to long term creep. In space the sag becomes negligible; creep under gravity also vanishes; the only dimensional change of this kind remaining is the delayed elastic effect due to the relaxation of stresses in the material. These can be eliminated in other ways.

The reduction in seismic disturbance also helps the Gyro Relativity experiment. The suspension torques include some terms proportional to the square of the acceleration, which rectify and cause noticeable drift errors from the vibrations of an Earth-based laboratory. In a free-flying spacecraft the periodic accelerations are greatly reduced.

The third potential advantage of space: the change in source of the effect to be measured does not affect the Gyro Relativity experiment unless one takes into consideration experiments about the Sun or Jupiter. It is hard to get close to the Sun. About Jupiter the magnitude of the geodetic precession is 100 arc-sec/year and of the motional precession 3.5

arc-sec/year. The only significant change between a ground-based laboratory and Earth orbit is the increase by a factor of 15 in the geodetic term through having a rotation period of 90 minutes rather than 24 hours.

For equivalence principle experiments and for Chapman's experiment on non-geodesic motion the merits of space are different. Operation in free fall is of only incidental importance; crucially important are the changes in source of the effect and the elimination of seismic noise.

Figure 3 illustrates the equivalence principle experiment proposed by P. W. Worden and C. W. F. Everitt. Two concentric cylinders of different materials--gold and aluminum, for example--

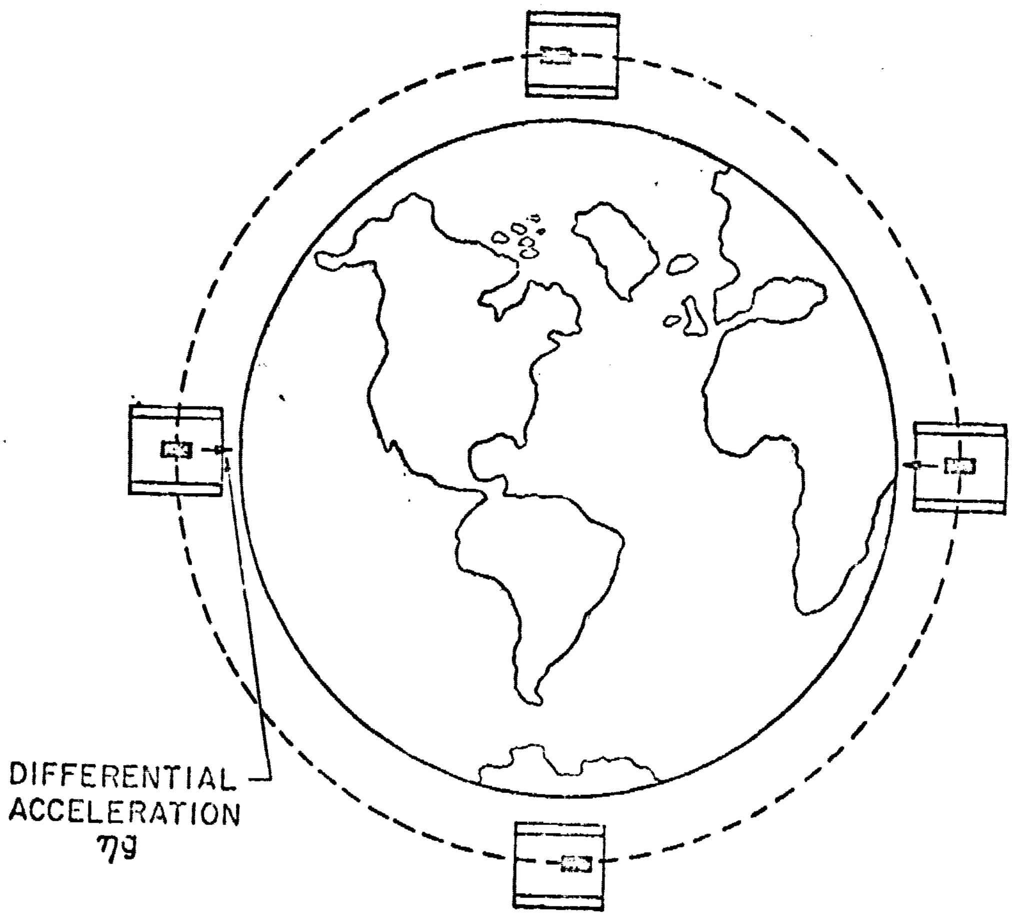

Figure 3: Concept of Orbital Equivalence Principle Experiment

orbit the Earth in a satellite held oriented in inertial space. The two masses are supported in superconducting bearings which constrain their lateral motions but leave them free to move along their common axis. Each is subject to the gravitational pull of the Earth, amounting in a 400 nautical mile orbit to 950 cm/sec^2, and the balancing orbital acceleration. Any difference between the ratios of gravitational to inertial mass for the materials will result in a periodic relative acceleration between the two masses along the common axis. The amplitude of the resultant relative motion depends on the natural periods of the masses along the axis. If the masses are essentially free floating so that their periods are long compared with the orbit period, then the amplitude corresponding to an Eötvös ratio η of 10^{-17} is 0.7 Å. The cryogenic techniques discussed in Section C allow this to be measured.

The driving acceleration f on an orbiting Eötvös experiment is 950 cm/sec^2, three orders of magnitude higher than the two sources available on Earth: the acceleration of 1.4 cm/sec^2 due to the Earth's rotation and the acceleration of 0.6 cm/sec^2 due to the attraction of the Sun. This then is one great advantage of operation in space. The other is the reduction of seismic vibration, which was the limiting factor in the experiment of Roll, Krotkov and Dicke. The analysis of Worden and Everitt indicates that it is indeed reasonable to expect an improvement of six orders of magnitude in the accuracy as compared with Earth-based experiments. The reasons for doing the experiment with two concentric masses, rather than simply flying a torsion balance are discussed in Section D.

Chapman's experiment on non-geodesic motion may be thought of as a variant equivalence principle experiment. General Relativity predicts that a spinning body does not exactly follow a geodesic when moving in the Reimannian space-time determined by the neighboring bodies. In other words a gyroscope in orbit experiences an anomalous acceleration, which has been calculated by Schiff for a spherically symmetric static field as

$$a_s = \frac{3GM}{mc^2R^5}\left[(\underline{R}\cdot\underline{H})(\underline{R}\wedge\underline{v}) + (\underline{R}\cdot\underline{v})(\underline{R}\wedge\underline{H})\right] \quad (4)$$

where m is the mass and $\underline{H}$ the angular momentum of the spinning body, $\underline{R}$ and $\underline{v}$ the orbit radius and orbit velocity. For a circular orbit $\underline{R}\cdot\underline{v} = 0$, the acceleration is directed along the orbit-normal and varies sinusoidally at orbit-frequency. If $\underline{H}$ lies in the orbit plane, the amplitude of the acceleration is

$$a_s = \frac{3H\Omega}{mc^2}\, g \quad (5)$$

where g is the local gravitational acceleration and Ω the orbital angular velocity.

Chapman suggested doing an experiment with a thin ring 5 m in diameter wound from fused silica fiber, spinning at 10,000 rpm, for which Equation (5) yields a periodic acceleration of 2.5×10^{-16} g. The amplitude of the periodic displacement relative to an independent non-spinning body at the center of the ring is $\pm$ 100 Å. A variant of the experiment suggested by C. W. F. Everitt, and investigated further by Chapman, is to compare two concentric counter-rotating rings of nearly equal radius. An experiment of this kind might indeed be feasible.

The advantages of space for the experiment on non-geodesic motion are slightly different from those of an ordinary equivalence principle experiment; they are indeed closer to those for the Gyro Relativity experiment. The effect is only fifteen times larger than it would be in a ground-based laboratory, but the experiment is simplified by the reduction in seismic vibration and the absence of the need to support the spinning bodies against gravity.

All three experiments described so far--the Gyro Relativity experiment, the equivalence principle experiment and the experiment on non-geodesic motion--are best done in a drag-free satellite: i.e., one in which drag forces are compensated by thrusters

on the spacecraft referenced to an internal proof mass. For the equivalence principle experiment the proof mass would be one of the test bodies for the experiment.

The experiments conceived by Braginsky, Caves and Thorne depend on the action of laboratory mass on a torsion balance. Figure 4, reproduced from the Braginsky-Caves-Thorne paper illustrates an idea for a "gravitational Ampère experiment." A spinning body $\bar{F}$ is suspended from one arm of a high-Q torsion balance, near which a large axially symmetric body M_s, of mass say one ton, is rotated with an angular velocity Ω_s, close to bursting speed, about the axis z of Figure 4(b). The spin-spin interaction between M_s and F causes a non-Newtonian attraction. By modulating the speed Ω_s at a frequency ω_0, $\Omega_s = \Omega_0 \cos\omega_0 t$ where the period $\tau_0 = 2\pi/\omega_0$ is equal to the natural frequency of the torsion balance (probably a few hours) the system is driven in resonance and the signal may be built up to a measurable level.

The difficulties of the gravitational Ampère experiment are discussed in Sections C and D. The advantages of space for it, and for a torsion balance experiment to measure the change in gravitational constant with time, is the absence of seismic vibration--and this may indeed be an absolute requirement for experiments of this kind. For reasons discussed in Section D both experiments would require an Earth-oriented spacecraft rather than one pointed in inertial space.

The most thorough investigation of a laboratory experiment to look for a change in the gravitational constant with time has been by R. L. Ritter of the University of Virginia, who is at present designing a version to be done on Earth. The experiment has also been discussed by Braginsky, Caves and Thorne. The rate of change that might reasonably be expected is G divided by the age of the Universe, or 1 part in 10^{11} per year. The idea is to measure the effect in a sophisticated Cavendish balance, and the principal difficulty is to find a way of calibrating the system that is stable to better than 1 part in 10^{11}

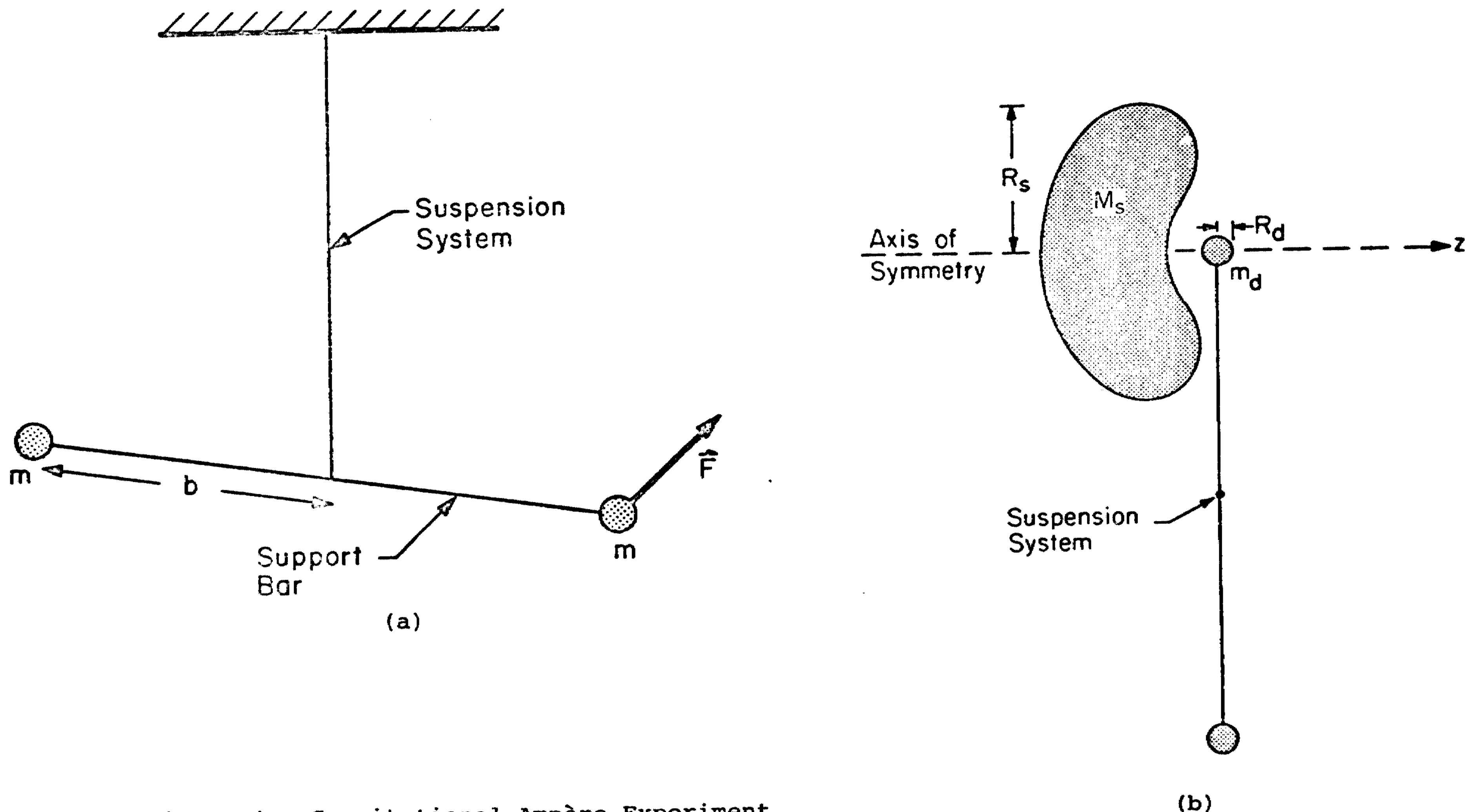

Figure 4: Gravitational Ampère Experiment

per year. Changes in mass, spring constant, dimension of the balance arm and so on might all masquerade as a change in G. An alternative way of measuring $\dot{G}/G$, first suggested by R. L. Forward, would be to orbit two bodies around each other in outer space and look for a change in orbital period.

A variety of other gravitational experiments have been proposed for which the advantages of space are slight or non-existent. The most obvious is the detection of gravitational waves by means of a Weber bar; others include experiments suggested by Braginsky, Caves and Thorne using vibrating or rotating laboratory masses to excite gravitational effects in high Q crystals or high Q superconducting cavities. Whenever the phenomena to be studied are at acoustic frequencies, as in the Weber bar, the seismic disturbances on Earth can be got rid of very effectively by the techniques developed by Weber and it is unlikely, at least for a very long time, that space will add anything.

A more promising long-term possibility for gravitational wave experiments in space is the suggestion made by a number of people of mounting laser interferometer detectors on a large space structure.

C. ADVANTAGES OF LOW TEMPERATURE OPERATIONS

(1) General

Four experiments were described in Section B that require measurements of extremely small accelerations or changes in accelerations on suspended bodies. These are (1) the equivalence principle experiment, (2) the experiment on non-geodesic motion, (3) the experiment on $\dot{G}/G$, (4) the Braginsky-Caves-Thorne "gravitational Ampère experiment." Each requires a mass or masses free from external disturbance and some means of measuring the linear or angular position of the suspended object. The Gyro Relativity experiment likewise requires very precise angular readout of the gyro spin axis. Low temperature techniques may be helpful in reducing the disturbances on the suspended body

or providing the readout. Before going into detailed reasons it is useful to tabulate and compare the magnitudes of the accelerations to be measured in the different experiments, using the typical experimental parameters stated in the last Section.

TABLE 1: Comparison of Accelerations to be Measured in Different Orbiting Gravitational Experiments

Experiment	Term to be Measured	Magnitude	Period
Equivalence Principle	$a_{\eta} = \eta g$	$10^{-17} g$	orbital
Non Geodesic Motion	$a_{\Omega} = \frac{3H\Omega}{mc^2}$	$2.5 \times 10^{-16} g$	orbital
$\dot{G}/G$	$\dot{a}_G = \frac{GM_O}{r^2} \frac{\dot{G}}{G}$	$10^{-17} g$/year	secular
"Gravitational Ampère" Effect	$a_A = \frac{GM_O}{5r^2} \frac{v^2}{c^2}$	$\sim 10^{-20} g$	resonant with torsion balanc

The size of the terms suggests that the first two will be the least difficult. Of course the equivalence principle experiment will become harder if the experimenters attempt a measurement of η to a higher accuracy than the current design goal of 1 part in 10^{17}.

(2) Fundamental Limits

The fundamental limit to determining the acceleration on a suspended body is thermal fluctuations. If the body supported is part of a lightly damped harmonic oscillator, for example a torsion balance or the superconducting bearing used for the equivalence principle experiment, it will be set vibrating with amplitude given (in the linear case) by

$$\langle \Delta x \rangle \sim \frac{2\pi}{\tau_o} \sqrt{\frac{kT}{M}} \qquad (6)$$

where M is the suspended mass and τ_o the natural period of the system. The oscillations tend to have the natural τ_o and persist in phase for roughly Q cycles where Q is the quality factor of the system. For a mass of 100g with a natural period of ten minutes the amplitude $\langle \Delta x \rangle$ is 23 Å--a factor of 30 higher than the 0.7 Å amplitude signal to be expected in an equivalence principle experiment at the 10^{-17} level, and some 30,000 higher than the signal to be expected in a "gravitational Ampère experiment."

The preceding paragraph describes the response of the system to thermal fluctuations, but does not fix a fundamental thermal noise limit to measurement of the acceleration of a suspended body. To determine that we must find the extent to which a particular signal can be resolved from noise: this depends on several factors, including noise introduced by back reaction from the position detector, noise in the position detector itself (readout noise), and the data-taking routine. We may assume that the data-taking routine approaches the optimum (as phase-sensitive detection for a signal of known phase and frequency), but the other factors are less under the control of the experimenters, and will be discussed below.

Assume for the moment that the reaction force on the suspended body and the noise in the position readout are negligible. The measurement is then limited by the fluctuation force, $\langle F \rangle$, determined from what is commonly known as the Nyquist formula, though it should be called the Einstein-Smoluchowski formula:

$$\langle F^2 \rangle = \frac{2M\beta kT}{S} \tag{7}$$

where β is the damping coefficient and S the time of observation. Putting F = Ma, with M the passive gravitational mass, the limit on an acceleration measurement in any of the four experiments described above is

$$\langle a \rangle \sim 1.4\sqrt{\frac{\beta kT}{MS}} \tag{8}$$

It is important to notice that the thermal limit on the acceleration measurement is independent of the natural period of the suspension system. It applies whether the suspended body is an isolated mass or an object suspended in a torsion balance, and contrary to what we might intuitively expect it is unaffected by making the driving acceleration resonant with the natural period of the suspension system. There may indeed be advantages in doing an experiment such as the Braginsky-Caves-Thorne "gravitational Ampère experiment" at resonance, but reduction in the fundamental thermal fluctuation noise is not one of them. The usual argument in favor of resonance is that it increases the amplitude of the signal and makes it easier to detect. In the presence of significant readout noise this may be the case; but if thermal noise is really the limiting factor, the data-taking and analysis schemes can have a larger effect than the condition of the resonance. The question of noise and back reaction from the position detector will be studied in Section C (3).

The quantity β in Equations (7) and (8) is the reciprocal of the natural decay time of oscillations of the suspension. Taking this as 10^8 sec (three years), then with M as 100 gm,

$$\langle a \rangle \sim 1.5 \times 10^{-16} \sqrt{\frac{T}{S}} \; g \tag{9}$$

With a temperature of 300K the limit on $\langle a \rangle$ is 4×10^{-17} g after an hour, 8×10^{-18} g after a day and 4.5×10^{-19} g after a year. With a temperature of 2K the limit on $\langle a \rangle$ is 3.5×10^{-18} g after an hour, 7×10^{-19} g after a day and 4×10^{-20} g after a year.

Thermal fluctuations then can easily be made small enough not to be a limit on any of the experiments described above except possibly the "gravitational Ampère experiment." Two caveats need to be entered, however. First, no one has ever actually done a gravitational experiment where thermal fluctuations proved a real limit. The nearest it has been approached has been in the room temperature equivalence principle experiments of Roll, Krotkov and Dicke and of Braginsky and Panov, in each of which the fundamental limit on η was about 10^{-13} in a day. The experiments lasted many days and reached limits of 3×10^{-11}

or slightly less. All talk of fundamental limits, even at room temperature, therefore, is at the moment academic. However since the practical limit in Roll, Krotkov and Dicke's work was seismic noise one may hope that space offers a real chance of coming closer to the fundamental limit.

The second point concerns the advantages of low temperature operation. Even if space does allow an experiment to reach the fundamental limit the advantage of low temperature operation is less than appears from Equation (8). With careful design the damping of any suspended system can be reduced to the point of being dominated by molecular exchange with the walls of the cavity. It is then proportional to gas pressure. There is, however, a limit below which the gas pressure cannot be reduced, and not just because of pumping difficulties. If there is any heat load on the suspended body, some means has to be provided for getting rid of the heat and at low temperatures the only way is by means of exchange gas. Now the damping on a suspended body of mass M attributable to gas at low pressure is given by

$$M\beta = CD^4\sqrt{\frac{2\pi m}{RT}}\, p \tag{10}$$

where m is the molecular weight of the gas molecule, p the pressure, D the characteristic dimension of the body, and C a constant depending on the shape and equal to $\frac{1}{12}$ for a sphere. Substituting in Equation (8) we reach the surprising conclusion that for a body of density ρ

$$\langle a\rangle \sim \frac{A}{\rho D}\; p^{1/2}\; T^{1/4} \tag{11}$$

Hence at constant pressure the advantages of going to low temperatures is only proportional to $T^{1/4}$: a meager gain.

With this we must turn to the problem of measuring the position of the suspended body.

(3) Problem of Position Measurement

The time-honored method of measuring the angular position of a body suspended from a torsion balance is the optical lever.

The limit on resolution of an optical lever is photon noise in the light falling on the mirror attached to the suspended system. For diffraction limited optics a straightforward calculation of the photon noise yields a limit on angular resolution

$$\delta\theta \sim 1.2 \times 10^{-11} \frac{1}{D^2} \quad \frac{\bar{\lambda}\bar{\nu}}{\phi\varepsilon} \tag{12}$$

where D is the diameter of the mirror, $\bar{\lambda}$ the color temperature of the lamp, ϕ the light flux, ε the total optical efficiency (including light losses in the system and the quantum efficiency of the photo detector), and $\bar{\nu}$ is the bandwidth of the detector. The best optical levers currently available, designed by R. V. Jones and J. C. S. Richards, attain photon noise limits and have an angular resolution of 10^{-5} arc-sec in a 10 Hz bandwidth -- well below the amplitude of the thermal fluctuations discussed in Section C (2) for any reasonable sized body at room temperature.

At low temperatures the use of an optical lever gives rise to a curious problem. As the temperature is lowered the thermal fluctuations get less and there comes a point where, other things being equal, the system is no longer limited by thermal fluctuations but rather by photon noise. To recover the ground one then has to increase the intensity of the light. Since some light is absorbed in the mirror, increasing the light intensity means increasing the heat input, and to carry away the extra heat it may be necessary to raise the gas pressure. One may then reach the paradoxical situation that a room temperature experiment will have lower thermal fluctuations than a cryogenic experiment, because the heat can be carried away radiatively at room temperature and the experiment can be operated at lower pressures. In fact the limit on the experiment at temperatures below about 80K (where radiative transfer loses effectiveness) is given by the sum of the squares of the thermal fluctuation and photon noise limits and has the form

$$\langle f^2 \rangle + \delta\gamma^2 = \frac{1}{D^2} \quad \left[\frac{\Lambda}{\rho^2}\, p + \frac{B}{T_o{}^4 p} \right] \frac{T}{S}^{1/2} \tag{13}$$

where A and B are constants, p the operating pressure, ρ, D and τ_o, density, dimension and natural period of the suspended system, T the temperature and S the time of observation. There is therefore an optimum working pressure, given by

$$p_{opt} = \text{const} \ \frac{\bar{\lambda}\Lambda}{\varepsilon\Delta T} \ \frac{\rho}{\tau_o^{\,2}} \tag{14}$$

where Λ is the absorption coefficient of the mirror and ΔT the maximum temperature difference allowed between the suspended body and its surroundings. Numerical calculation shows that at a pressure of 10^{-9} torr the period of a typical suspended system has to exceed one day to prevent domination by photon noise. At 10^{-11} torr the period would have to exceed 10 days.

To do the ultimate experiment, then, it is no good simply to take a standard torsion balance, put it at low temperatures, pump to low pressures and hope for the best. Some form of position readout more suitable than the optical lever is needed.

A good possibility is the superconducting position readout developed by P. W. Worden, Jr., and myself for the orbiting equivalence principle experiments, and independently by H. Paik for the Stanford Gravity Wave Antenna. This depends on measuring a change in magnetic field due to the motion of the body by means of a SQUID (Superconducting QUantum Interference Device) magnetometer and superconducting circuits enclosed in a superconducting magnetic shield. The superconducting test body is placed between two coils in which an external field has been trapped. The superconductor is a perfect diamagnet, and its motion causes a redistribution of field in the coils which may be detected by the magnetometer. Figure 5 shows the details. A pair of superconducting coils, each having inductance L_0, are joined in a continuous loop, with a third inductance L_2 in parallel with them. A persistent current flows through the main loop, and if the current in L_2 is initially zero, a motion of the superconducting test mass modulates the inductances L_0 so that, by flux conservation, a current ΔI_2 flows through L_2 given by

$$\Delta I_2 \approx 2I \frac{L_0}{L_0 + 2L_2} \frac{\Delta x}{y} \tag{15}$$

where Δx is the displacement of the mass, y is the distance from the coils L_0 to the mass, and I is the persistent circulating current, assumed to be much greater than I_2. Note that, by symmetry, the current in L_2 is insensitive to changes in length of the test mass. The current I_2 is read out by a SQUID detector, which then drives a centering servo. A readout for relative position of two masses follows the same principle but has one coil adjacent to each mass for a differential measurement.

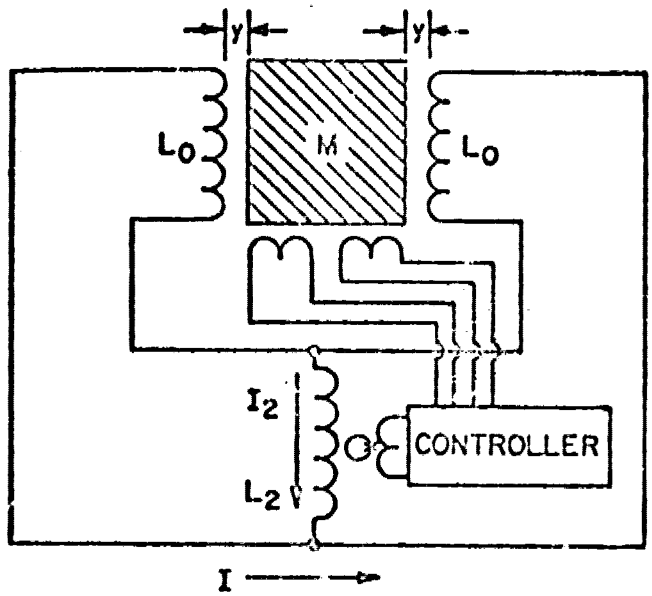

Figure 5: Position Detector for Equivalence Principle Experiment

The readouts exert reaction forces on the test masses, which for the idealized case of a single body illustrated in Figure 5, makes the system a simple harmonic oscillator with a natural frequency ω_0 given by

$$\omega_0^2 = \frac{2I^2 L_0^2}{(L_0 + 2L_0)My^2} \tag{16}$$

where M is the mass of the test body. The dynamics of the full system of coupled bodies is complicated. It is advantageous to design the apparatus so that the relative position sensor provides the main restoring force. The system can then be approximated

as a simple harmonic oscillator with one degree of freedom in which the resonant frequency is again given by ω_0, with L_0 and L_2 referring to the relative position sensor and M being the reduced mass of the two bodies.

For the quantum interference detector connected to the circuit of Figure 5, what we have called readout noise determines the minimum detectable displacement δx. The quantity δx depends on the circuit parameters, the quantity of flux trapped in the circuit and the fractional resolution of the flux quantum ϕ_0 by the detector. Combining the expression for δx with Equation (6) we find the ratio of readout noise to thermal motions for a system where the restoring force is wholly due to the trapped current in the readout coils,

$$\frac{\delta x}{\langle \Delta x \rangle} = \varepsilon(\tau)\,\frac{\phi_0}{L_2}\sqrt{\frac{L_0 + 2L_2}{kT}} \qquad (17)$$

where $\varepsilon(\tau)$ is the fractional resolution of the flux quantum and L_0 and L_2 are the inductances illustrated in Figure 5. Thus the ratio $\delta x/\langle \Delta x \rangle$ is independent of the masses and of the natural period of the system. For a system with characteristic dimension 10 cm and $\varepsilon(\tau)$ taken as 10^{-4} the numerical value of $\delta x/\langle \Delta x \rangle$ at 2K is about 10^{-3}. In other words the readout magnetometer tracks the position of the test masses with great accuracy and makes no significant contribution to the total experimental error. The limit is then indeed set by the fundamental formula, Equation (8).

(4) Practical Advantages of Cryogenic Techniques

The argument of Section C (2) shows that low temperature operation is not strictly necessary to reduce the fundamental limits to the level needed to do any of the four acceleration experiments except possibly the "gravitational Ampère experiment," and furthermore that the advantage in thermal fluctuations at low temperature is less than one might suppose, being proportional only to $T^{1/4}$. What then are the real grounds, if any, for choosing cryogenic techniques in experiments of this kind?

There seem to be five:

(i) readout. The quantum interference detector supplies a convenient, precise, linear and stable position readout.

(ii) magnetic shielding. A superconducting shield around the apparatus protects the readout and proof masses adequately against disturbances from changes in the external field, which conventional room temperature iron shields cannot do.

(iii) mecnanical stability. At room temperature a change of 0.01°C in the temperature of a fused quartz apparatus 10 cm long will make it expand by 6 Å, ten times the calculated displacement for free masses with an Eötvös ratio of 10^{-17}. Cyclic temperature variations at orbital frequency could masquerade as a violation of the equivalence principle. At low temperatures such effects are completely negligible. Creep is also reduced at low temperature.

(iv) effects of residual gas in the experimental chamber. If the satellite moves parallel to the common axis of the two proof masses, gas molecules in the experimental chamber transfer momentum in different amounts to the two masses, making them accelerate differentially. A motion periodic with the orbit will create a spurious Eötvös signal. Probably this effect could be made small even at room temperature. At a pressure of 10^{-8} torr the amplitude of the periodic component of the satellite motion would need to be less than 0.1 mm, which should be achievable with a drag-free satellite. However the requirement is relaxed an order of magnitude at cryogenic temperatures.

A similar advantage is obtained with regard to noise from certain gas streaming effects, which may be due to outgassing or to thermal gradients; these produce pressure differences which can disturb a sensitive accelerometer. Outgassing is enormously reduced for all surfaces except those contaminated with helium. A thermal gradient may cause pressure differences because molecules moving from the hot side carry more momentum than those from the cold side. The difference, in first order, is independent of temperature and depends only on the temperature gradient. But the relaxation time for thermal gradients at low temperatures scales as T^2 for metals, and superfluid helium is an essentially perfect heat conductor. It is possible to make the temperature almost perfectly uniform in a dewar in the presence of a substantial external heat load. The corresponding gas pressure effects are reduced many orders of magnitude.

(v) effects of black body radiation in the experimental chamber. Black body radiation exerts pressure on the test mass. At room temperature the unbalanced pressure at the end of a right cylinder of density 10 would cause an acceleration of 1.3 x 10^{-9} g, five orders of magnitude larger than the Eötvös acceleration for two masses with an η of 10^{-17}. Hence cyclically changing temperature gradients of 0.001°C across the experimental chamber could masquerade as an Eötvös signal of 2 x 10^{-17}. At low temperatures all such effects are reduced by several orders of magnitude.

D. EFFECTS OF GRAVITY GRADIENTS

Critical to any experiment on gravitation in space is the effect of gravity gradients. In the Gyro Relativity experiment a torque acts on the gyroscope owing to the interaction of its quadrupole mass moment with the gradient of the Earth's gravitational field. If the gyro spin axis is misaligned with the orbit plane by an angle α the result is a secular drift

$$\Omega^g = \frac{3}{4}\,\frac{\Delta I}{I}\,\frac{1}{\omega_s}\,\frac{GM}{R^3}\,\sin 2\alpha \tag{18}$$

similar to the secular drift of the Earth's axis from the Sun's gravity gradient which gives rise to the precession of the equinoxes. Equation (18) may be transposed into an upper limit on the allowable inertia-ratio $\Delta I/I$ for a gyroscope to yield a given drift performance. For a gryo required to have a drift-rate below 10^{-16} rad/sec (6 x 10^{-4} arc-sec) in the worst case (where α is 45°), $\Delta I/I$ would have to be less than 10^{-7}. For a gyro in near polar orbit with its spin axis within 1° of the orbit plane $\Delta I/I$ could be allowed to be as high as 3 x 10^{-6}. In practice the allowed limit can be slightly higher than 3 x 10^{-6} since techniques exist for evaluating the magnitude of the term and calculating it out. Whatever the exact limit, the gravity gradient term like the mass-unbalance term described in Section B, imposes a severe restriction on the allowable density variations in the ball.

Gravity gradient effects are even more important in the design of a space-borne equivalence principle experiment.

The difference in acceleration on two masses separated by a distance ΔR along the radius vector R from a central body of mass M is $2GM\Delta R/R^3$ and the corresponding torque on a torsion balance with a residual quadrupole mass moment is

$$\Gamma^g = \frac{3}{2} J_2 I \frac{GM}{R^3} \sin 2\theta \tag{19}$$

where I is the moment of inertia of the balance, J_2 the coefficient of the quadrupole term and θ the angle between the quadrupole and the direction to the accelerating body. In the ground-based equivalence experiments of Dicke and Braginsky this torque is barely noticeable since the gradient of the Sun's field is small, but in an orbital experiment it becomes very important, for the Earth's gravity gradient is more than 10^7 times larger than the Sun's. Although the gradient torque on a torsion balance can in principle be separated out from an Eötvös torque since it is of twice orbital frequency, whereas the Eötvös effect occurs at orbital frequency, the difficulties in making a complete separation are formidable. For a ring of radius r and mass M constructed in two halves of different materials with Eötvös ratio η, the Eötvös torque from a driving acceleration f is $1/2\eta fMr$. Since the moment of inertia of the ring is Mr^2 and CM/R^3 is equal to f/R we have for the ratio of the two torques

$$\frac{\Gamma_{\text{gravity gradient}}}{\Gamma_{\text{Eötvös}}} = \frac{3J_2 r}{\eta R} \tag{20}$$

Consider an extremely well balanced test body for which J_2 is 10^{-4} and assume one is trying to measure η to 10^{-17}; if r is 10 cm the gravity gradient torque is 4.6×10^5 times larger than the Eötvös torque. It is difficult to enhance the Eötvös signal relative to the gravity gradient torque by resonating the suspension with the orbital period. For large Q's, the relative enhancement of the response to the fundamental to the response to the second harmonic is less than 3Q. The time required to approach this equilibrium situation is about Q cycles: about three months for a Q of 1000. At the

end of that time the Eötvös signal is enhanced by about 1000 -- and the gravity gradient signal is at least 150 times larger. Regardless of the patience of the experimenters, there remains the difficulty that small nonlinearities in the system or its readout can cause frequency mixing of such a large second harmonic with third or higher harmonics, to produce a singly periodic signal looking like an Eötvös signal. The spectrum of gravity gradient noise has a rich harmonic structure, particularly if the orbit is eccentric.

Besides any frequency mixing due to non-linearities there is a subharmonic in the gravity gradient torque on a torsion balance due to the ellipticity of the orbit. This has been evaluated by Mark Zimmermann. If ψ is the angle between the axis of the balance, R the orbit major axis, and ε the eccentricity there are sine and cosine components identical in period with the Eötvös torque Γ^E:

$$\Gamma_s^g = \frac{3}{8} J_2 I \frac{GM}{R^3} \varepsilon \cos 2\psi \sin \omega t \tag{21}$$

$$\Gamma_c^g = \frac{3}{8} J_2 I \frac{GM}{R^3} \varepsilon \sin 2\psi \cos \omega t \tag{22}$$

Even though the phases of Γ_s^g and Γ_c^g may be expected to be different from Γ^E we have here a real limit on the experiment since the direction of J_2 and of the major axis of the orbit become progressively harder to fix the smaller ε and J_2 are. Probably the limit from this source on determining η from a torsion balance in space would be something like 10^{-14} or 10^{-15} at best.

These difficulties are avoided in the experiment of Worden and Everitt by measuring the relative linear displacement of two nearly coincident freely falling masses, for example, two coaxial cylinders (see Figure 3 above). If the ratio of gravitational to inertial mass for the two bodies is not quite equal there will be a differential acceleration between them which will cause

a periodic relative displacement along the cylinder axis at orbital frequency. If the masses are truly in free fall the amplitude of their relative motion is $T^2 \eta g'/4\pi^2$ where T is the orbit period and g' is the local gravitational acceleration (about 950 cm/sec^2 at 300 nautical miles altitude). With a 100 minute period and an η of 10^{-17}, the amplitude of the relative displacement is 0.6 Å. The essence of the experiment is measuring the periodic component of the displacement at orbital frequency to determine η. The ratio of the gravity gradient acceleration to the Eötvös acceleration in this case is $2\Delta R/R\eta$. It can be made unity if $\Delta R \sim R\eta \sim 7 \times 10^8\ \eta$ cm; so to measure η to 10^{-17} the average value of ΔR should be about 1 Å. Such accurate centering is achieved by using the doubly periodic gravity gradient signal as a dither signal. The amplitude and phase of the second harmonic acceleration is a direct measure of the projection on the orbit plane of the center of mass displacement ΔR. The procedure, then, is to detect the second harmonic in the output and drive it to null with a servo loop that moves the masses. Although there is an orbital frequency subharmonic in the free fall experiment also due to the ellipticity of the orbit, it, being of relative order $\varepsilon\Delta R/R\eta$, is completely negligible.

Both torsion-balance and free fall experiments are subject to gravity gradient disturbances (which may be labelled "gravity gradient noise") from the motions of nearby masses, for example, movements of the spacecraft or an astronaut, or tidal slosh in the liquid helium for a cryogenic experiment. P. W. Worden, Jr. has extended the comparison of free fall and torsion balance equivalence principle experiments by forming expressions for the higher order gravity gradient terms on the free-fall cylinders and comparing the ratios (S/N) of a given Eötvös signal S to gravity gradient disturbances N for the two experiments. Provided the cylinders are centered enough to make the first order term negligible, the ratio of the two (S/N)s is

$$\frac{(S/N)_{\text{free fall}}}{(S/N)_{\text{torsion balance}}} \sim \frac{\varepsilon\xi^2}{JDR} \tag{23}$$

where R is the distance to the disturbing mass, ξ the radius of gyration of the outer cylinder in the free-fall experiment, and ε the quantity which when multiplied by the principal moment of the cylinder gives the difference between its largest and smallest principal moments. For given manufacturing procedures applied to the two classes of experiment the quantities J_2 and ε are comparable, as are the characteristic dimensions ξ and D for the two apparatuses. Hence the ratio of the two (S/N)s is of order ξ/R. So far as concerns the Earth's gravity gradient the advantage of the free-fall experiment over the torsion balance is between six and eight orders of magnitude. For disturbance sources on the spacecraft it is between one and two orders of magnitude. The crucial problem in an experiment mounted on a free-flying spacecraft is tidal sloshing of the helium. Taking ℓ as the length of an optimized outer cylinder and z as the mean distance to the helium, the mass amplitude M_H of the helium tide in a experiment to measure η to a limiting precision η_0 must satisfy the inequality

$$M_H < \frac{gz^6}{G\ell^4}\,\eta_0 \tag{24}$$

With ℓ around 10 cm M_H must be less than 20 gm for a 10^{-17} experiment. The corresponding tidal amplitude is 1 mm. Several techniques are available to control slosh. Since M_H scales as z^6/ℓ^4 there is great advantage in making a large cavity around a relatively small apparatus.

The equivalence principle experiment has to be done in a spacecraft whose orientation is fixed in inertial space. Experiments such as the measurement of $\dot{G}/G$ or the attempt to measure a "gravitational Ampère term" do not use the Earth as a source and can therefore be performed in an Earth-oriented

spacecraft with the torsion balance axis aligned to the local vertical. In this case gravity gradient disturbances from the Earth are independent of time, and the use of torsion balances is practical. There remain some smaller effects due to eccentricity of the orbit and the resulting periodic misalignment of the axis with the vertical, and gravity gradient forces from the satellite body and massive rotor. The experimenters plan to use special highly symmetric configurations to reduce the Newtonian gravity gradient forces from the rotor, and similiar techniques applied to the satellite do the rest. The advantage of space operation is, as stated in Section B, the elimination of seismic disturbance.

Gravity gradient effects are also extremely important in the experiment on non-geodesic motion, chiefly with respect to alignment and centering of the two spinning bodies. The problem here is that the signal is perpendicular to the orbit plane, rather than in-plane as in the equivalence principle experiment. An offset of the centers of mass in the perpendicular direction leads to a periodic displacement with orbital frequency because the masses follow independent, intersecting orbits. This displacement will generally have a component at the phase of the signal from non-geodesic effects. Any problems from miscentering in the orbit plane can be removed by the dither technique of the equivalence principle experiment. Gravity gradients from the satellite body will also be a source of noise here as in the gravitational ampere and equivalence principle experiment.

In addition to the above difficulties, gravity gradient forces cause torques on the rotors and hence a slow precession, as in the Gyro Relativity experiment. Ordinarily this precession will be very slow because of the large angular momentum and small torque, and should cause no fundamental problem. However, precession may be useful in another sense: if it is large enough it will modulate the non-geodesic motion term and allow it to be distinguished from a displacement along the orbit axis. Other tricks can possibly be played with the

pointing of the rotor axes and the eccentricity of the orbit. Practical problems may be foreseen with the position readout for two precessing rings, but practical problems are always the worst difficulty of any of these experiments.

E. RECOMMENDATIONS

The Gyro Relativity experiment is reasonably well established as a NASA SRT program. Some laboratory research is currently in progress on developing superconducting bearings for the free fall equivalence principle experiment. While one should always keep an open mind to fresh possibilities it seems very unlikely that a torsion balance equivalence principle experiment can even compete with it.

The experiment on non-geodesic motion deserves further study and so probably does the possibility of developing an experiment to measure $\dot{G}/G$. The gravitational Ampère experiment presents an altogether higher order of difficulty since the magnitude of the effect to be measured is at least three orders of magnitude smaller and the difficulties of supporting a laser a spinning body from a high Q torsion balance and rotating a one-ton mass near it at bursting speed are horrendous.

Gravitational wave experiments with a Weber bar in space appear to have little merit. There is plenty of room for further progress on Earth first and no real promise of improvement from space operations. The use of large space structures for interferometer gravity wave detector is more interesting. However the caution should be expressed that no one has made a working detector of this kind yet on Earth, and the claims by various people to be just on the verge of success incorporate a fairly high optimism factor.